

Hong Kong Mathematics Olympiad (1994 – 95)

Final Event 6 (Group)

香港數學競賽 (1994 – 95)

決賽項目 6 (團體)

$2^a \cdot 9^b$ is a four digit number and its thousands digit is 2, its hundreds digit is a , its tens digit is 9 and its units digit is b , find a , b .

$a =$

$2^a \cdot 9^b$ 為一四位數，其千位數是 2，百位數是 a ，十位數是 9，個位數是 b ，求 a 及 b 。

$b =$

Find c , if $c = \left(1 + \frac{1}{2} + \frac{1}{3}\right)\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) - \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right)\left(\frac{1}{2} + \frac{1}{3}\right)$.

若 $c = \left(1 + \frac{1}{2} + \frac{1}{3}\right)\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) - \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right)\left(\frac{1}{2} + \frac{1}{3}\right)$ ，求 c 。

$c =$

Find d , if

$$d = \left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{1994}\right)\left(\frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{1995}\right) - \left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{1995}\right)\left(\frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{1994}\right).$$

求 d ，若

$$d = \left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{1994}\right)\left(\frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{1995}\right) - \left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{1995}\right)\left(\frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{1994}\right)。$$

$d =$

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Final Event 7 (Group)

香港數學競賽 (1994 – 95)

決賽項目 7 (團體)

- (i) Let p, q, r be the three sides of triangle PQR . If

$$p^4 + q^4 + r^4 = 2r^2(p^2 + q^2)$$

find a , where $a = \cos^2 R$ and R denotes the angle opposite r .

設 p, q, r 為三角形 PQR 的三邊。若

$$p^4 + q^4 + r^4 = 2r^2(p^2 + q^2)$$

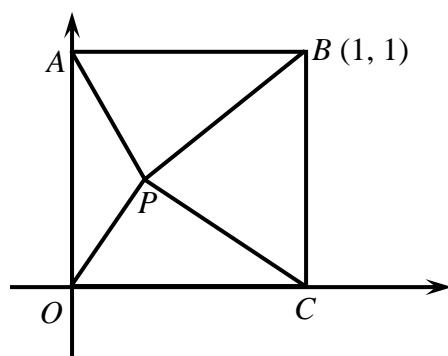
且 $a = \cos^2 R$ 其中 R 的對邊為 r ，求 a 。

$a =$

- (ii) Refer to the diagram, P is any point inside the square $OABC$ and b is the minimum value of $PO + PA + PB + PC$, find b .

$b =$

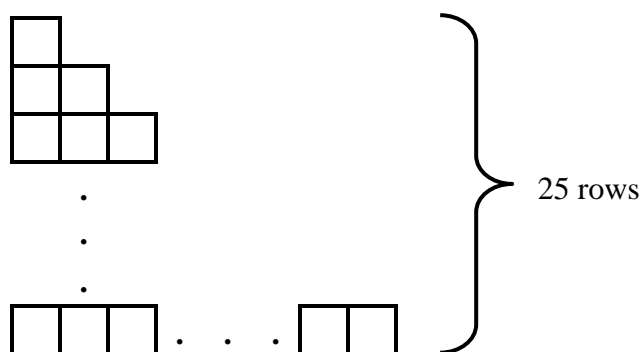
如圖， P 為正方形 $OABC$ 內的任意點，且 b 為 $PO + PA + PB + PC$ 之最小值，求 b 。



- (iii) Identical matches of length 1 are used to arrange the following pattern, if c denotes the total length of matches used, find c .

$c =$

長度同為 1 的火柴被排成下列圖案。若以 c 表示用去火柴枝的總長，求 c 。



- (iv) Find d , where $d = \sqrt{111111 - 222}$.

$d =$

求 d ，若 $d = \sqrt{111111 - 222}$ 。

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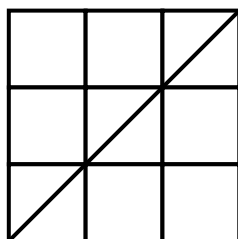
Final Event 8 (Group)

香港數學競賽 (1994 – 95)

決賽項目 8 (團體)

Rectangles of length ℓ and breadth b where ℓ, b are positive integers, are drawn on square grid paper. For each of these rectangles, a diagonal is drawn and the number of vertices V intersected (excluding the two end points) is counted (see figure) .

在方格紙上繪畫尺寸為 $\ell \times b$ 的長方形，其中 ℓ, b 為正整數並添上對角線一條。以 V 代表相交的端點總數 (不包括首尾兩點在內)。(如圖示)



$$\ell = b = 3$$

$$V = 2$$

- (i) Find V , when $\ell = 6, b = 4$.

$$V =$$

當 $\ell = 6, b = 4$ 時，求 V 。

- (ii) Find V , when $\ell = 5, b = 3$.

$$V =$$

當 $\ell = 5, b = 3$ 時，求 V 。

- (iii) When $\ell = 12$ and $1 < b < 12$, find r , the number of different values of b that makes $V = 0$?

$$r =$$

當 $\ell = 12$ 且 $1 < b < 12$ 時，求使 $V = 0$ 時， b 的不同個數 r 。

- (iv) Find V , when $\ell = 108, b = 72$.

$$V =$$

當 $\ell = 108, b = 72$ 時，求 V 。

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Final Event 9 (Group)

香港數學競賽 (1994 – 95)

決賽項目 9 (團體)

A, B, C, D are different integers ranging from 0 to 9 and

$$\begin{array}{r} A A B C \\ - B A C B \\ \hline D A C D \end{array}$$

Find A, B, C and D.

A、B、C、D 為由 0 至 9 間的不同整數，且

$$\begin{array}{r} A A B C \\ - B A C B \\ \hline D A C D \end{array}$$

求 A、B、C 及 D 之值。

A =

B =

C =

D =

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Final Event 10 (Group)

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決賽項目 10 (團體)

Lattice points are points on a rectangular coordinate plane having both x - and y -coordinates being integers. A moving point P is initially located at $(0, 0)$. It moves 1 unit along the coordinate lines (in either directions) in a single step.

在直角坐標平面上， x -和 y -坐標同為整數的點稱為格點。 P 是起始時位於 $(0, 0)$ 的移動點，它每一步必須沿坐標線的其中一過個方向走 1 個單位的距離。

- (i) If P moves 1 step then P can reach a different lattice points, find a .

$a =$

若 P 走 1 步，它可到達 a 個格點，求 a 。

- (ii) If P moves not more than 2 steps then P can reach b different lattice points, find b .

$b =$

若 P 可走不超過 2 步，它可到達 b 個格點，求 b 。

- (iii) If P moves 3 steps then P can reach c different lattice points, find c .

$c =$

若 P 走 3 步，它可到達 c 個格點，求 c 。

- (iv) If d is the probability that P lies on the straight line $x + y = 9$ when P advances 9 steps, find d .

$d =$

若 P 走 9 步，它停在直線 $x + y = 9$ 上的概率是 d ，求 d 。

